Enrollment No: _____ Exam Seat No:_____

C.U.SHAH UNIVERSITY
Summer Examination-2019

Subject Name: Graph Theory

Subject Code: 4SC06GRT1 **Branch: B.Sc. (Mathematics)**

Semester: 6 **Date: 29/04/2019** **Time: 10:30 To 01:30** **Marks: 70**

Instructions:
(1) Use of Programmable calculator & any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

Q-1 Attempt the following questions: **(14)**

a) How many edges in 6-regular graph with 9 vertices? (01)

b) Draw a diagraph with 7 vertices in which three vertices have 2 degree, two vertices have 4 degree and remaining vertices have 6 degree. (02)

c) Define: Complete graph (02)

d) Define: Ring sum of two subgraphs (02)

e) True/False: K_6 is a Euler graph. (01)

f) Is C_n a Hamiltonian graph? (01)

g) Vertex connectivity of any graph is always _____to the edge connectivity of *G*. (01)

h) What is the rank of $A(G)$? (01)

i) True/False: Adjacency matrix is a symmetric matrix. (01)

j) State Dirac's theorem. (02)

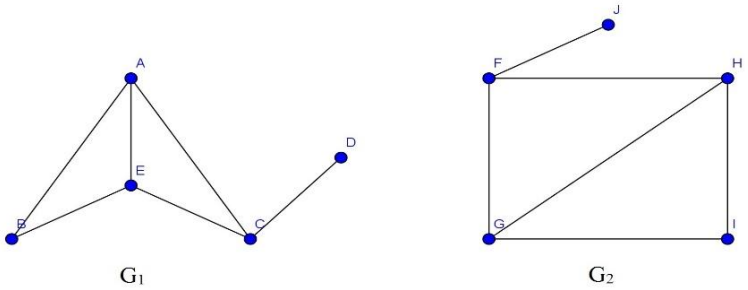
Attempt any four questions from Q-2 to Q-8

Q-2 Attempt all questions **(14)**

a) State and prove first theorem of graph theory and also verify it for K_4. (05)

b) Define Degree sequence and draw a graph with degree sequence 0,1,1,2,3,4,5. (05)

c) Show that the following graphs are isomorphic. (04)



G_1 G_2

Figure – 1

Q-3 Attempt all questions **(14)**

a) In a complete graph with n vertices, there is $\frac{n-1}{2}$ edge disjoint Hamiltonian circuits if $n > 3$ and n is also an odd. (05)

b) Find center, radius and diameter of the following graph. (05)

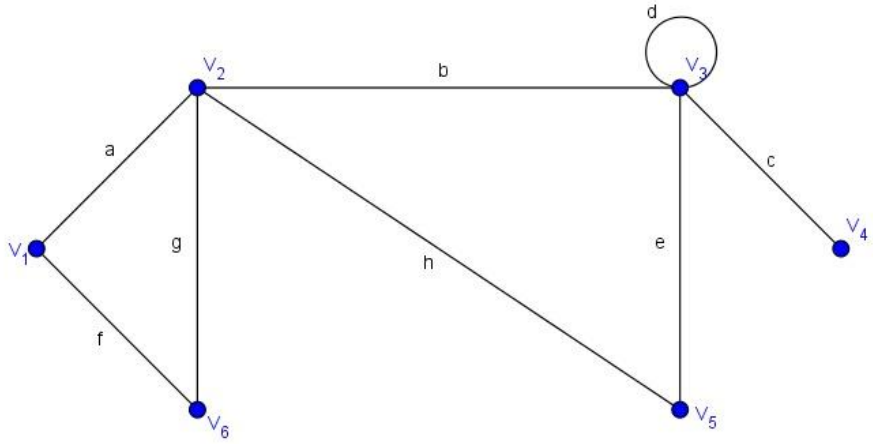


Figure – 2

c) Answer the following for graph which shows in figure-3. (04)

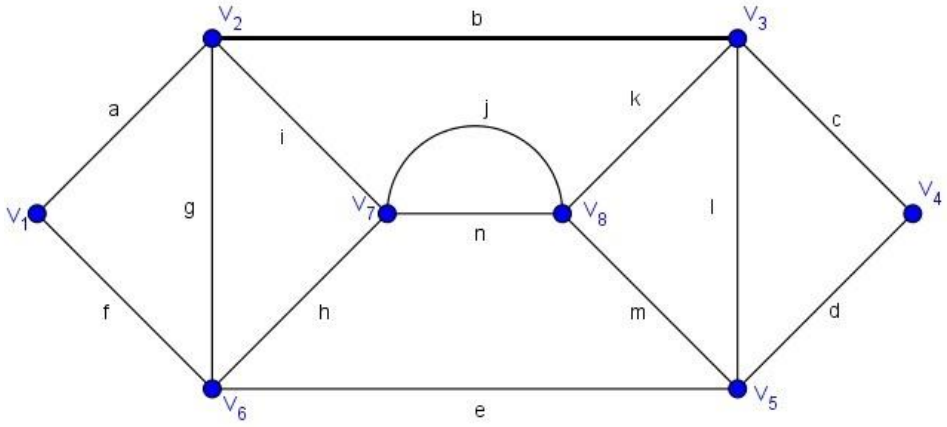


Figure – 3

i) Write degree of each vertex.
ii) How many odd and even vertices?
iii) Write one path of length 7.
iv) Write one closed walk of length 9.

Q-4 Attempt all questions **(14)**

a) Let G be a simple graph with n vertices and k-components then G have at most $\frac{(n-k)(n-k+1)}{2}$ number of edges. (07)

b) State and prove Euler's theorem. (07)

Q-5 Attempt all questions **(14)**

a) State and prove necessary and sufficient condition for the graph is disconnected. (07)

b) Prove that every tree has either one or two centers. (04)

c) Define: Decomposition graph, Fusion, Hamiltonian circuit (03)

Q-6 Attempt all questions **(14)**

a) Explain Konigsberg bridge problem and write the solution given by Euler. (05)

b) If G be a tree with n is vertices then prove that it has $(n-1)$ edges. (05)

c) Define binary tree and also find the number of pendent vertices in binary tree with n vertices. (04)

Q-7 Attempt all questions **(14)**

a) Answer the following questions from the figure-4: (07)

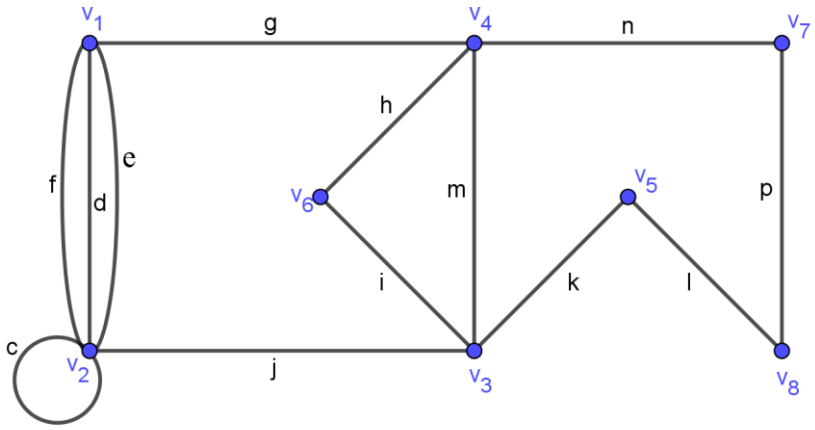


Figure-4

i) Write one spanning tree.

ii) Write three fundamental cut-sets w.r.t. i).

iii) Write one fundamental circuit w.r.t. i).

iv) How many branches and chords are in this graph?

v) What is the vertex and edge connectivity of this graph?

b) Define: Connected graph, Spanning subgraph, Cycle, Spanning tree (04)

c) Find the adjacency matrix for the following figure-5: (03)

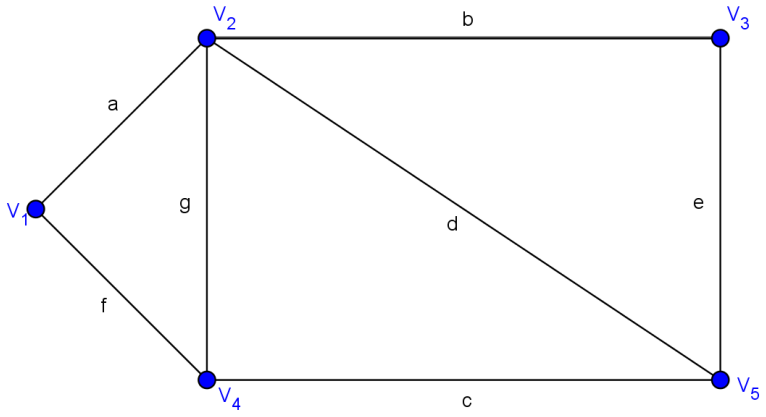


Figure – 5

Q-8 Attempt all questions **(14)**

a) Verify $AB^T = O$ for the figure-5, where A and B are incidence matrix and circuit matrix respectively. (07)

b) Find the path matrix $P(V_2, V_3)$ for figure-5. (05)

c) Define: Cut-set matrix (02)